

# Can the quintessence be a complex scalar field?

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## Abstract

In light of the recent observations of type Ia supernovae suggesting an accelerating expansion of the Universe, we wish in this paper to point out the possibility of using a complex scalar field as the quintessence to account for the acceleration. In particular, we extend the idea of Huterer and Turner in deriving the reconstruction equations for the complex quintessence, showing the feasibility of making use of a complex scalar field (instead of a real scalar field) while maintaining the uniqueness feature of the reconstruction for two possible situations respectively. We discuss very briefly how future observations may help to distinguish the different quintessence scenarios, including the scenario with a positive cosmological constant.

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# 1 Introduction

The Supernova Cosmology Project<sup>1</sup> and the High-Z Supernova Search<sup>2</sup> reported on their observations of type Ia supernovae (SNe Ia), suggesting that the expansion of the Universe is still accelerating [1, 2]. In addition, recent measurements of the power spectrum of the cosmic microwave background (CMB) from BOOMERANG [3] and MAXIMA-1 [4] detected a sharp peak around  $l \simeq 200$ , indicating that the Universe is flat. Combining these two classes of observations, we may conclude that the Universe has the critical density (to make it flat) and that it consists of 1/3 of ordinary matter and 2/3 of dark energy with negative pressure (such that  $p_{total} < -\rho_{total}/3$  at present time) [5, 6]. At the moment, the most often considered candidates for the dark energy include (1) the existence of a positive cosmological constant [7] and (2) the presence of a slowly-evolving real scalar field called "quintessence" [8]. In the case of the quintessence, Zlatev, Wang and Steinhardt [9] considered a real scalar field as the "tracker field" which, regardless of a wide range of possible initial conditions, will join a path more or less common to the evolving radiation, the dominant energy density in the early universe, before the matter-dominated era. In addition, Huterer and Turner [10] considered basically the inverse problem of reconstructing the quintessence potential from the SNe Ia observational data (see also Refs. [11] and [12]).

In this paper, we wish to point out the possibility of using a complex scalar field as the quintessence to account for the accelerating expansion of the Universe. In particular, we extend the idea of Huterer and Turner in deriving the reconstruction equations for the complex quintessence, showing the feasibility of making use of a complex scalar field (instead of a real scalar field) while maintaining the uniqueness of the solution of the inverse problem. We also discuss briefly how future observations may help to distinguish the different quintessence scenarios (including the scenario with a positive cosmological constant).

## 2 The Basics

We consider a spatially flat Universe which is dominated by the non-relativistic matter and a spatially homogeneous complex scalar field  $\Phi$  and which is described by the flat Robertson-Walker metric:

$$ds^2 = dt^2 - a^2(t) (dr^2 + r^2 d\varphi_1^2 + r^2 \sin^2 \varphi_1 d\varphi_2^2). \quad (1)$$

The action for the Universe is

$$S = \int d^4x \sqrt{g} \left( -\frac{1}{16\pi G} \mathcal{R} - \rho_M + \mathcal{L}_\Phi \right), \quad (2)$$

where  $g$  is the absolute value of the determinant of the metric tensor  $g_{\mu\nu}$ ,  $G$  is the Newton's constant,  $\mathcal{R}$  is the Ricci scalar,  $\rho_M$  is the matter density, and  $\mathcal{L}_\Phi$  is the Lagrangian density for the complex scalar field  $\Phi$ :

$$\mathcal{L}_\Phi = \frac{1}{2} g^{\mu\nu} (\partial_\mu \Phi^*) (\partial_\nu \Phi) - V(|\Phi|), \quad \mu, \nu = 0, 1, 2, 3. \quad (3)$$

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<sup>1</sup><http://snap.lbl.gov>

<sup>2</sup><http://cfa-www.harvard.edu/cfa/oir/Research/supernova/HighZ.html>

In Eq. (3), we have assumed that the potential  $V$  depends only on the absolute value (or the amplitude) of the complex scalar field:  $|\Phi|$ .

Instead of  $\Phi$  and  $\Phi^*$ , we would like to use the alternative field variables: the amplitude  $\phi(x)$  and the phase  $\theta(x)$  (of the complex scalar field  $\Phi(x)$ ), which are defined by

$$\Phi(x) = \phi(x)e^{i\theta(x)}. \quad (4)$$

(More precisely,  $\Phi(t) = \phi(t)e^{i\theta(t)}$ .) The usage of the field variables— $\phi(x)$  and  $\theta(x)$ —will benefit the derivation of the reconstruction equations which relate the quintessence potential  $V(\phi)$  to SNe Ia data. By using Eq. (4), the Lagrangian density for  $\Phi$  (Eq. (3)) becomes

$$\mathcal{L}_\Phi = \frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) + \frac{1}{2}\phi^2g^{\mu\nu}(\partial_\mu\theta)(\partial_\nu\theta) - V(\phi). \quad (5)$$

The variation of the action (Eq. (2)) with the above Lagrangian density yields the Einstein equations and the field equations of the complex scalar field. By using the metric tensor in Eq. (1), these equations can be rearranged and become

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}\left[\rho_M + \left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\phi^2\dot{\theta}^2\right) + V(\phi)\right] \quad (6)$$

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G}{3}(\rho + 3p) = -\frac{8\pi G}{3}\left[\frac{1}{2}\rho_M + \left(\dot{\phi}^2 + \phi^2\dot{\theta}^2\right) - V(\phi)\right] \quad (7)$$

$$\ddot{\phi} + 3H\dot{\phi} - \dot{\theta}^2\phi + V'(\phi) = 0 \quad (8)$$

$$\ddot{\theta} + \left(2\frac{\dot{\phi}}{\phi} + 3H\right)\dot{\theta} = 0, \quad (9)$$

where  $H$  is the Hubble parameter, dot and prime denote derivatives with respect to  $t$  and  $\phi$  respectively,  $\rho$  is the energy density, and  $p$  is the pressure. We note that the non-relativistic matter contributes the energy density  $\rho_M$  and pressure  $p_M = 0$ , while the evolving complex scalar field contributes the energy density  $\rho_\Phi$  and pressure  $p_\Phi$  as follows:

$$\rho_\Phi = \frac{1}{2}\left(\dot{\phi}^2 + \phi^2\dot{\theta}^2\right) + V(\phi) \quad (10)$$

$$p_\Phi = \frac{1}{2}\left(\dot{\phi}^2 + \phi^2\dot{\theta}^2\right) - V(\phi). \quad (11)$$

Eqs. (6)–(9) are the fundamental equations which govern the evolution of the Universe.

We first note that Eq. (9) can be solved and the solution for the “angular velocity”  $\dot{\theta}$  is given by

$$\dot{\theta} = \frac{\omega}{a^3\phi^2}, \quad (12)$$

where  $\omega$  is an integration constant determined by the initial condition of  $\dot{\theta}$  (or the value of  $\dot{\theta}$  at some specific time). By using Eq. (12), the fundamental equations become

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}\left[\rho_M + \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\frac{\omega^2}{a^6}\frac{1}{\phi^2} + V(\phi)\right] \quad (13)$$

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G}{3}(\rho + 3p) = -\frac{8\pi G}{3}\left[\frac{1}{2}\rho_M + \dot{\phi}^2 + \frac{\omega^2}{a^6}\frac{1}{\phi^2} - V(\phi)\right] \quad (14)$$

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\omega^2}{a^6}\frac{1}{\phi^3} + V'(\phi) = 0. \quad (15)$$

Eq. (15) can be rearranged and become

$$\ddot{\phi} + 3H\dot{\phi} + \frac{d}{d\phi}\left[\frac{1}{2}\frac{\omega^2}{a^6}\frac{1}{\phi^2} + V(\phi)\right] = 0. \quad (16)$$

The term  $\omega^2/(2a^6\phi^2)$  in the bracket, coming from the “angular motion” of the complex scalar field  $\Phi$ , can be treated as an effective potential (to be called “centrifugal potential”). It produces a “centrifugal force” and tends to drive  $\phi$  away from zero if  $\omega \neq 0$  (i.e. the “angular velocity”  $\dot{\theta}$  is nonzero).

We note that, because the gravitational influence of the dark energy with negative pressure tends to prevent the ordinary matter from forming structures, the contributions from quintessence to the energy density and pressure should have been insignificant just a short time ago. Furthermore, for preserving the concordance between theories (Big Bang Cosmology + Inflation + Cold Dark Matter) and observations (for instance, measurements of CMB and light element abundances), the contributions from quintessence should also be negligible at the epochs of ‘recombination’ and ‘primordial nucleosynthesis’. This constraint on quintessence will help to disentangle two kinds of kinetic energy provided by the complex quintessence, one from evolving  $\phi$  and the other from the “angular motion” of the complex scalar field  $\Phi$ , as follows.

In Eqs. (13) and (14), the contributions from the “angular motion” of  $\Phi$  to the energy density  $\rho$  and pressure  $p$  both are proportional to  $a^{-6}\phi^{-2}$ . The factor  $a^{-6}$  may make these contributions decrease very fast, even faster than the matter density  $\rho_M$  (which is proportional to  $a^{-3}$ ), provided that  $\phi$  does not decrease as fast as  $a^{-3/2}$  (i.e. along with the expansion of the Universe). In this situation, the angular-motion contributions are negligible at the present epoch, since they should have been insignificant just a short time ago. On the other hand, under the situation that  $\phi$  decreases no slower than  $a^{-3/2}$ , the angular-motion contributions to the energy density and pressure are no longer necessary to be negligible, while the contributions from evolving  $\phi$  (proportional to  $\dot{\phi}^2$ ) in Eqs. (13) and (14) would decrease no slower than  $a^{-3}H^2$ . In this situation, the evolving- $\phi$  contributions can be neglected at the present epoch, since they should also have been insignificant a short time ago.

To sum up, we have two possible situations, depending on how fast  $a^{-6}\phi^{-2}$  falls off, compared with  $\rho_M$  (i.e.  $a^{-3}$ ), as the Universe expands. One situation is that the contributions from the “angular motion” of the complex scalar field  $\Phi$  to the energy density and pressure are negligible. The other is that the contributions from evolving  $\phi$  are negligible. Indeed the disentanglement of these two types of kinetic energy in these two situations plays a crucial role in making the reconstruction of the quintessence potential feasible, as will be discussed in the next section. In addition, we note that these two possible situations are corresponding to two kinds of quintessence. The one with negligible angular-motion

contributions behaves like the real quintessence. The other with negligible evolving- $\phi$  contributions may be distinct from the real quintessence if the angular-motion contributions are significant. When both the contributions from evolving  $\phi$  and the “angular motion” of the complex scalar field  $\Phi$  are negligible such that the quintessence potential dictates, it is equivalent to the case of having a positive cosmological constant.

### 3 Reconstruction Equations

The quantities introduced in Sec. 2

$$\begin{aligned} t & : \text{Robertson-Walker time coordinate} \\ a(t) & : \text{scale factor} \\ H(t) & : \text{Hubble parameter} \\ \rho_M(t) & : \text{matter energy density} \end{aligned} \tag{17}$$

are neither observable quantities themselves in experiments, nor directly related to observations. In order to obtain the reconstruction equations for the quintessence potential  $V(\phi)$ , we need to use the observationally relevant quantities

$$\begin{aligned} z & : \text{redshift} \\ r(z) & : \text{Robertson-Walker coordinate distance} \\ & \quad \text{to an object at redshift } z \\ H_0 & : \text{Hubble constant} \\ \Omega_M & : \text{matter energy density fraction} \end{aligned} \tag{18}$$

to replace them. The quantities in (17) and (18) are related by

$$1 + z = \frac{1}{a} \tag{19}$$

$$r(z) = - \int_{t_0}^{t(z)} \frac{dt'}{a(t')} = \int_0^z \frac{dz'}{H(z')} \tag{20}$$

$$H(z) = \frac{\dot{a}}{a} = \frac{1}{(dr/dz)} \tag{21}$$

$$\rho_M = \Omega_M \rho_c = \frac{3\Omega_M H_0^2}{8\pi G} (1 + z)^3, \tag{22}$$

where  $\rho_c$  is the critical density, and we have set the present scale factor  $a_0$  to be one. In addition, for the quantity  $\ddot{a}/a$  in Eq. (14), which is related to the acceleration of the expansion, we have

$$\frac{\ddot{a}}{a} = \frac{1}{(dr/dz)^2} + (1 + z) \frac{d^2 r/dz^2}{(dr/dz)^3}. \tag{23}$$

Using Eqs. (19)–(23), we can obtain the reconstruction equations from the fundamental equations (13) and (14). These reconstruction equations relate  $V(\phi)$ ,  $\dot{\phi}$ , and  $\dot{\theta}$  to the observationally relevant quantities  $z$ ,  $r(z)$ ,  $H_0$ , and  $\Omega_M$  as follows:

$$V[\phi(z)] = \frac{1}{8\pi G} \left[ \frac{3}{(dr/dz)^2} + (1 + z) \frac{d^2 r/dz^2}{(dr/dz)^3} \right] - \frac{3\Omega_M H_0^2}{16\pi G} (1 + z)^3, \tag{24}$$

$$\left(\frac{d\phi}{dz}\right)^2 + \frac{\omega^2}{\phi^2}(1+z)^4 \left(\frac{dr}{dz}\right)^2 = \frac{(dr/dz)^2}{(1+z)^2} \left[ -\frac{1}{4\pi G} \frac{(1+z)(d^2r/dz^2)}{(dr/dz)^3} - \frac{3\Omega_M H_0^2}{8\pi G} (1+z)^3 \right], \quad (25)$$

where the term  $(\omega^2/\phi^2)(1+z)^4(dr/dz)^2$  in Eq. (25) is the contribution from the “angular motion” of the complex scalar field  $\Phi$ .

Through the reconstruction equations, given the data  $r(z)$  from SNe Ia experiments and the values of the parameters  $\Omega_M$  and  $H_0$  obtained from other experiments, it seems that we can reconstruct the quintessence potential  $V(\phi)$  after inputting the values of  $\omega$  and  $\phi_0$ , which correspond to some specific initial conditions. But, unlike the case of a real scalar field discussed in [10], the values of  $\omega$  and  $\phi_0$  are not the parameters we can input arbitrarily. They will determine the proportion of the contributions from the “angular motion” of the complex scalar field  $\Phi$  to the energy density and pressure. As discussed in Sec. 2, we have two possible situations, the one in which the contributions from the “angular motion” of the complex scalar field  $\Phi$  are negligible and the other one in which the contributions from evolving  $\phi$  are negligible. In the following, however, we will show that the reconstruction of  $V(\phi)$  is still possible for both situations.

For the situation in which the angular-motion contributions are negligible, i.e.  $\phi$  does not decrease as fast as  $a^{-3/2}$ , the reconstruction equations become the same as those in [10] as follows:

$$V[\phi(z)] = \frac{1}{8\pi G} \left[ \frac{3}{(dr/dz)^2} + (1+z) \frac{d^2r/dz^2}{(dr/dz)^3} \right] - \frac{3\Omega_M H_0^2}{16\pi G} (1+z)^3 \quad (26)$$

$$\left(\frac{d\phi}{dz}\right)^2 = \frac{(dr/dz)^2}{(1+z)^2} \left[ -\frac{1}{4\pi G} \frac{(1+z)(d^2r/dz^2)}{(dr/dz)^3} - \frac{3\Omega_M H_0^2}{8\pi G} (1+z)^3 \right]. \quad (27)$$

We note that in this case only the shape of  $V(\phi)$  and the corresponding region of  $V(\phi)$  (which corresponds to the observational region of redshift  $z$ ) can influence the evolution of the Universe, while the value of  $\phi_0$  has no influence. Thus, in this case, the initial value  $\phi_0$  can be put in by hand, and  $\omega$  is no longer the parameter we need to input. With the initial value  $\phi_0$ , the parameters:  $\Omega_M$  and  $H_0$ , and the data  $r(z)$ , we can obtain the information on  $V(z)$  and  $\phi(z)$  through the reconstruction equations (26) and (27), respectively, and then reconstruct the quintessence potential  $V(\phi)$  for some specific region of  $V(\phi)$  corresponding to the observational region of redshift  $z$ . We note that the reconstruction of  $V(\phi)$  in this case is in the same way as the case of a real scalar field discussed in [10].

On the other hand, for the situation in which the evolving- $\phi$  contributions are negligible, i.e.  $\phi$  decreases no slower than  $a^{-3/2}$ , the reconstruction equations become

$$V[\phi(z)] = \frac{1}{8\pi G} \left[ \frac{3}{(dr/dz)^2} + (1+z) \frac{d^2r/dz^2}{(dr/dz)^3} \right] - \frac{3\Omega_M H_0^2}{16\pi G} (1+z)^3 \quad (28)$$

$$\left(\frac{\omega}{\phi}\right)^2 = \frac{1}{(1+z)^6} \left[ -\frac{1}{4\pi G} \frac{(1+z)(d^2r/dz^2)}{(dr/dz)^3} - \frac{3\Omega_M H_0^2}{8\pi G} (1+z)^3 \right]. \quad (29)$$

With the initial value  $\phi_0$ , the parameters:  $\Omega_M$  and  $H_0$ , and the data  $r(z)$ , Eq. (29) may be used to determine the value of  $\omega$  and yield the information on  $\phi(z)$ , and Eq. (28) gives

the information on  $V(z)$  in the observational region of redshift  $z$ . Then the reconstruction of the quintessence potential  $V(\phi)$  can be achieved for some region of  $V(\phi)$  corresponding to the observational region of redshift  $z$ . Unlike the previous case, the reconstruction of  $V(\phi)$  in this case is different from the case of a real scalar field discussed in [10], because the possible existence of a significant angular-motion contribution here is the very feature that the real quintessence does not possess.

As shown in the above, two viable quintessence potentials may be reconstructed, with the use of SNe Ia data, for two possible situations respectively. Nevertheless, the self-consistency of the reconstruction through Eqs. (28) and (29), for the situation in which the evolving- $\phi$  contributions are negligible, should be checked, as follows. From Eq. (29), we can obtain  $\phi(z)$ , and also  $\phi(a)$ . We then check whether  $\phi(a)$  fits the requirement that the evolving- $\phi$  contributions (proportional to  $\dot{\phi}^2$ ) are negligible at the present epoch. If  $\phi(a)$  does not pass this consistency check, the complex quintessence model with significant angular-motion contributions is ruled out, and we can only use Eqs. (26) and (27) to reconstruct the quintessence potential instead. Likewise, for the situation in which the angular-motion contributions are negligible, we can check whether  $\phi(a)$ , obtained from Eq. (27), fits the ‘sufficient condition’ of this situation:  $\phi$  does not decrease as fast as  $a^{-3/2}$ . If not, the reconstruction of the quintessence potential through Eqs. (26) and (27) will be accompanied by a delicate choice of a small enough “angular-velocity” parameter  $\omega$ , therefore possessing a naturalness problem.

## 4 Discussion and Summary

In this work, we have investigated the scenario of using a complex scalar field as the quintessence for accelerating the expansion of the Universe. In the present scenario, there are two kinds of “kinetic-energy” type contributions to the energy density and pressure, one coming from the evolving amplitude  $\phi$  and the other from the “angular motion” of the complex scalar field  $\Phi$ . In many cases, the contribution from the “angular motion” may decrease very fast along with the expansion of the Universe, and is negligible in the process of reconstructing the quintessence potential  $V(\phi)$  (which is responsible for the possible accelerating expansion of the Universe). Nevertheless, there is also a situation in which the contribution from the evolving amplitude  $\phi$  is negligible while the part from the “angular motion” need to be treated with care.

Making use of the reconstruction equations (24) and (25) (as derived from the fundamental equations (6)–(9)), we may reconstruct the quintessence potential  $V(\phi)$  from the observational data  $r(z)$  (the coordinate distance as a function of the redshift  $z$ , as may be deduced from SNe Ia experiments), respectively for the two situations mentioned above. Accordingly, the complex scalar fields may be used as the candidate for the quintessence and our analysis indicates that, depending on how fast  $a^{-6}\phi^{-2}$  falls off as the Universe expands, the quintessence potential may be reconstructed in a fairly unique manner.

It is useful to note that the observation data on  $r(z)$  may be converted uniquely into the information on the effective equation of state of the dark energy:  $w(z) \equiv p_X(z)/\rho_X(z)$  (where ‘X’ denotes the dark energy) [12, 13]. Such information may in turn be used to distinguish different quintessence scenarios: The scenario with a positive cosmological constant corresponds to  $w = -1$  so that a significant variation of  $w$ , especially differing from

the value of  $-1$ , would help to rule out such scenario. The distinction between the real and complex scalar field scenarios is obviously more subtle: When the “angular motion” part is negligible, the complex quintessence behaves like the real one. However, the situation when the “angular motion” part is more important by comparison needs to be further studied since it poses new possibilities for the Universe.

In any event, the complex scalar fields as the quintessence should be seriously considered since such fields, unlike the real scalar field, have been invoked in many different sectors of elementary particle physics, such as the possibility for gauging (i.e., interacting with the various gauge fields), responsible for mass generations (Higgs mechanisms), as well as arising from condensates into Goldstone “pions” (from techni-color quark-antiquark pairs). In our opinion, the rich physics associated with the complex fields should be taken seriously in constructing a workable model, or theory, for the early universe.

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